

$$V = \mathbb{R} \quad F = \mathbb{Q}$$

$$x, y, z \quad \alpha, \beta, \gamma$$

- 1) $x + y \in V$
- 2) $x, y, z \in V$
- $(x + y) + z = x + (y + z)$
- 3) $x + \underline{0} = x = 0 + x$
- 4) $x + \underline{-x} = 0$
- 5) $x + y = y + x$
- 6) $\alpha x \in V \quad \alpha \in F, x \in V$
- 7) $(\alpha\beta)x = \alpha(\beta x)$
- 8) $\underline{1} \cdot x = x = x \cdot 1 \quad 1 \in F$
- 9) $\alpha(x + y) = \alpha x + \alpha y$
- 10) $(\alpha + \beta)x = \alpha x + \beta x$

closure property w.r.t. '+'

Associativity w.r.t. +

'0' Additive identity

$-x$ Additive Inverse

commutativity w.r.t. +

closure w.r.t. '.'

scalar multiplication

'1' multiplicative identity

} Distributive laws.

Eg. 2) $V = \mathbb{R} \quad F = \mathbb{R}$

x, y, z α, β, γ

values scalars

$V(F) \rightarrow$ is called vector space if it satisfies above (10) properties.

Eg. $\mathbb{R}(\mathbb{Q})$, $\mathbb{R}(\mathbb{R})$

Eg. 3) $V \rightarrow 2 \times 2 \text{ Matrices:}$ $F \rightarrow \mathbb{R}$

A, B, C $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ α, β, γ

i) $A, B \in V \quad A + B \in V$

$$2) (A+B) + C = A + (B+C) \quad A, B, C \in V$$

$$3) A + \underline{0} = A \quad 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$4) A + \underline{-A} = \underline{0}$$

$$5) A + B = B + A$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

$$= \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$= \begin{bmatrix} e+a & f+b \\ g+c & h+d \end{bmatrix}$$

$$6) \alpha A \in V$$

$$7) (\alpha\beta)A = \alpha(\beta A)$$

$$8) 1A = A \quad 1 \in \mathbb{R}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$9) \alpha(A+B) = \alpha A + \alpha B$$

$$\alpha \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right\} = \alpha \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$= \begin{bmatrix} \alpha a + \alpha e & \alpha b + \alpha f \\ \alpha c + \alpha g & \alpha d + \alpha h \end{bmatrix}$$

$$= \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{bmatrix} + \begin{bmatrix} \alpha e & \alpha f \\ \alpha g & \alpha h \end{bmatrix}$$

$$= \alpha A + \alpha B$$

$$10) (\alpha + \beta)A = \alpha A + \beta A$$

$$(\alpha + \beta) \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$(V, +, \cdot)$$

$$\dots \dots \dots$$

$$F = \mathbb{C}$$

$$\mathbb{C} \rightarrow \text{complex NO.}$$

$\{ c \ a \}$
H.W. $V = C$ $F = C$ $C \rightarrow \text{complex NO.}$
 $C(C) \rightarrow \text{V.S.}$ $(a+ib) + (c+id) = (a+c) + i(b+d)$
 $(a+ib)(c+id) = (ac-bd) + i(ad+bc)$

$C(R)$

Q. $Q(R)$ is a v.s. ?? $R(C)$??

Note:

